

# Lattice study of dense two-color QCD

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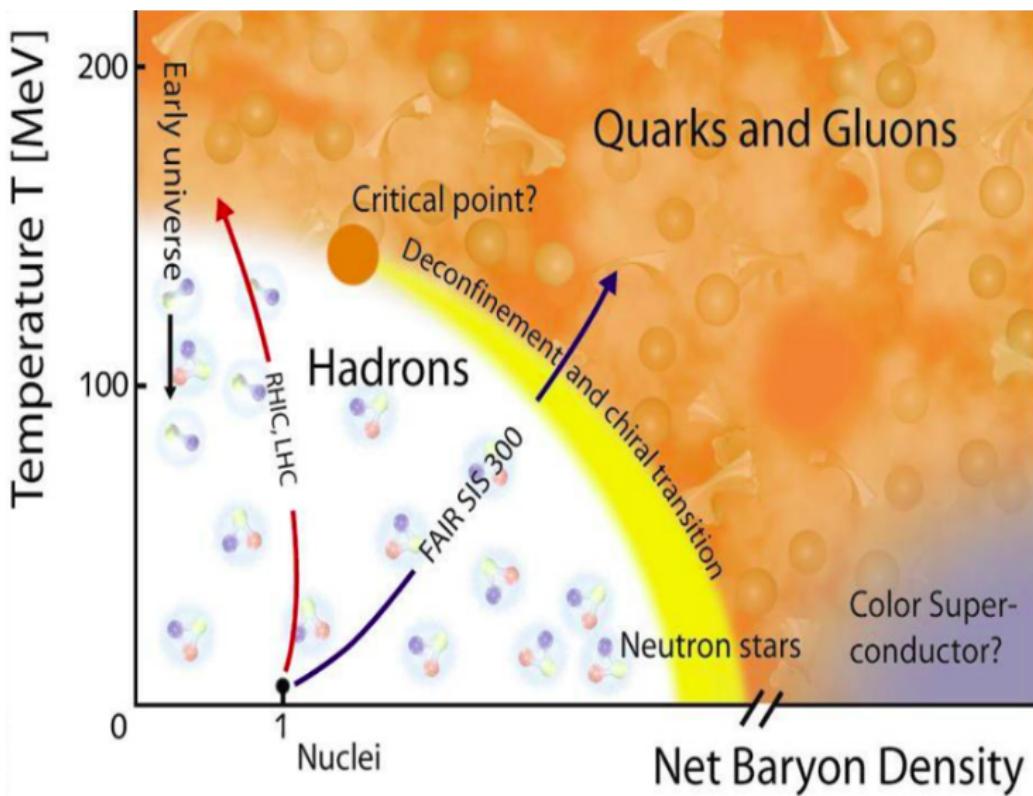
based on arXiv:1605.04090, Phys. Rev. D 94, 114510 (2016)



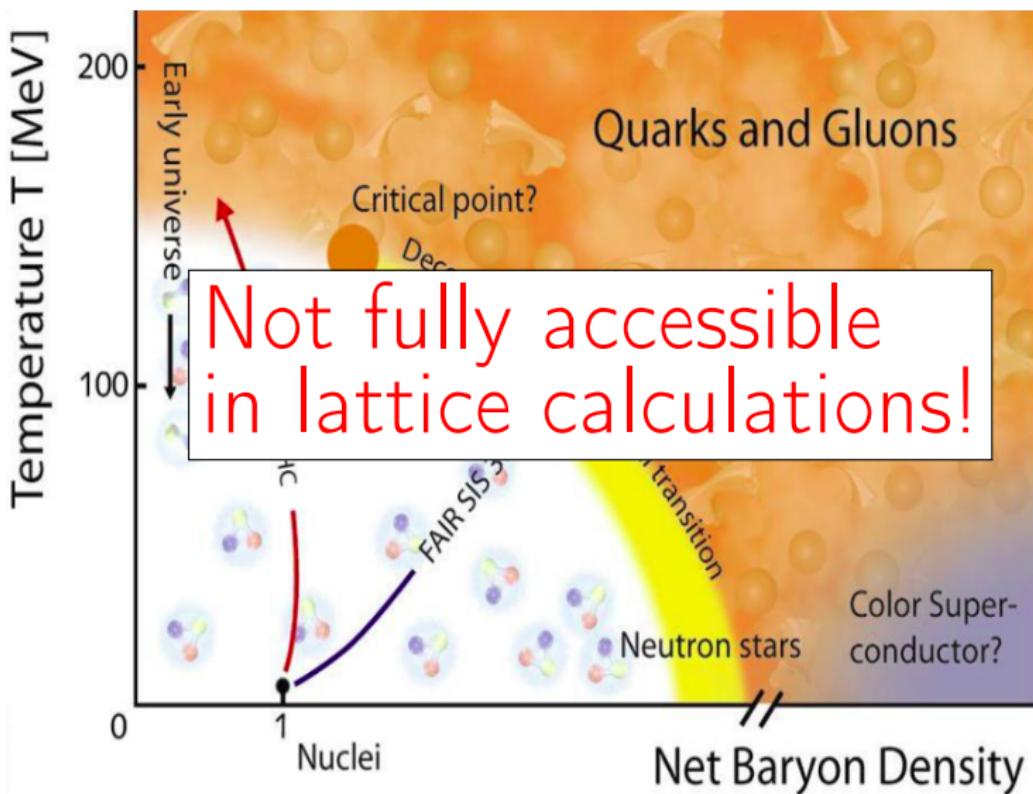
QCD in Finite Temperature and Heavy-Ion Collisions

13 February, 2017

## QCD phase diagram



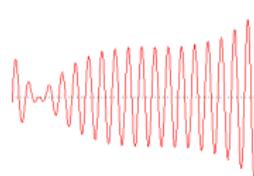
## QCD phase diagram



# Sign problem

## SU(3) QCD

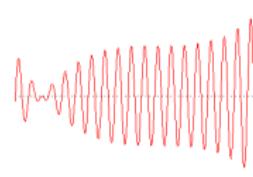
- ▶  $Z = \int DUD\bar{\psi}D\psi \exp(-S_G - \int d^4x \bar{\psi}(\hat{D} + m)\psi) = \int DU \exp(-S_G) \times \det(\hat{D} + m)$
- ▶ Eigenvalues go in pairs  $\hat{D} : \pm i\lambda \Rightarrow \det(\hat{D} + m) = \prod_{\lambda} (\lambda^2 + m^2) > 0$   
i.e. one can use lattice simulation
- ▶ Introduce chemical potential:  $\det(\hat{D} + m) \rightarrow \det(\hat{D} - \mu\gamma_4 + m) \Rightarrow$  the determinant becomes complex (sign problem)



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## SU(2) QCD

- ▶  $(\gamma_5 C\tau_2) \cdot D^* = D \cdot (\gamma_5 C\tau_2)$
- ▶ Eigenvalues go in pairs  $\hat{D} - \mu\gamma_4$ :  $\lambda, \lambda^*$
- ▶ For even  $N_f$   $\det(\hat{D} - \mu\gamma_4 + m) > 0 \Rightarrow$  free from sign problem

## Differences between $SU(3)$ and $SU(2)$ QCD

- ▶ The Lagrangian of the  $SU(2)$  QCD has the symmetry:  
 $SU(2N_f)$  as compared to  $SU_R(N_f) \times SU_L(N_f)$  for  $SU(3)$  QCD
- ▶ Goldstone bosons ( $N_f = 2$ )  $\pi^+, \pi^-, \pi^0, d, \bar{d}$

## Similarities:

- ▶ There are transitions: confinement/deconfinement, chiral symmetry breaking/restoration
- ▶ A lot of observables are very close:

**Topological susceptibility** (*Nucl.Phys.B*715(2005)461):

$$\chi^{1/4}/\sqrt{\sigma} = 0.3928(40) \text{ } (SU(2)), \quad \chi^{1/4}/\sqrt{\sigma} = 0.4001(35) \text{ } (SU(3))$$

**Critical temperature** (*Phys.Lett.B*712(2012)279):

$$T_c/\sqrt{\sigma} = 0.7092(36) \text{ } (SU(2)), \quad T_c/\sqrt{\sigma} = 0.6462(30) \text{ } (SU(3))$$

**Shear viscosity** :

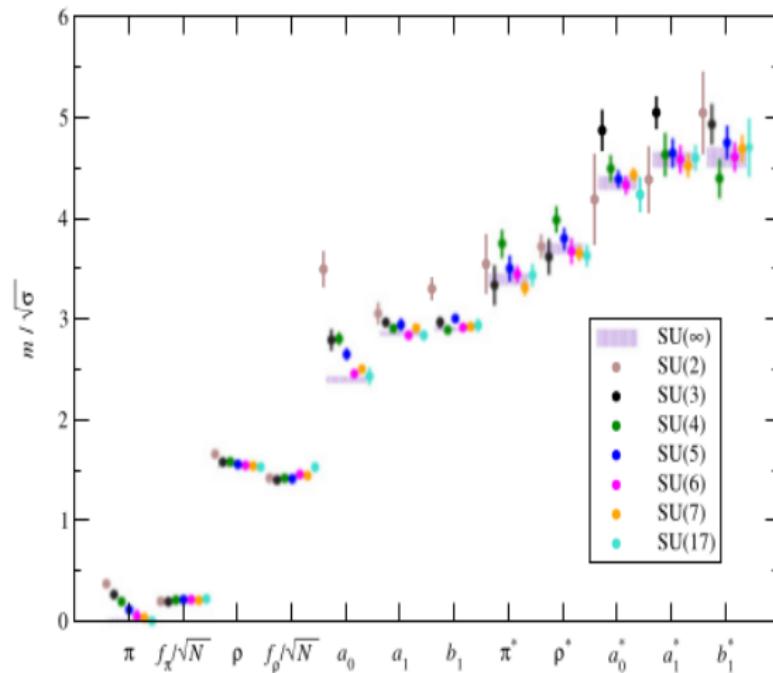
$$\eta/s = 0.134(57) \text{ } (SU(2)), \quad \eta/s = 0.102(56) \text{ } (SU(3))$$

JHEP 1509(2015)082

Phys.Rev. D76(2007)101701

## Similarities:

- ▶ Spectroscopy (Phys.Rep.526(2013)93)



## To summarize:

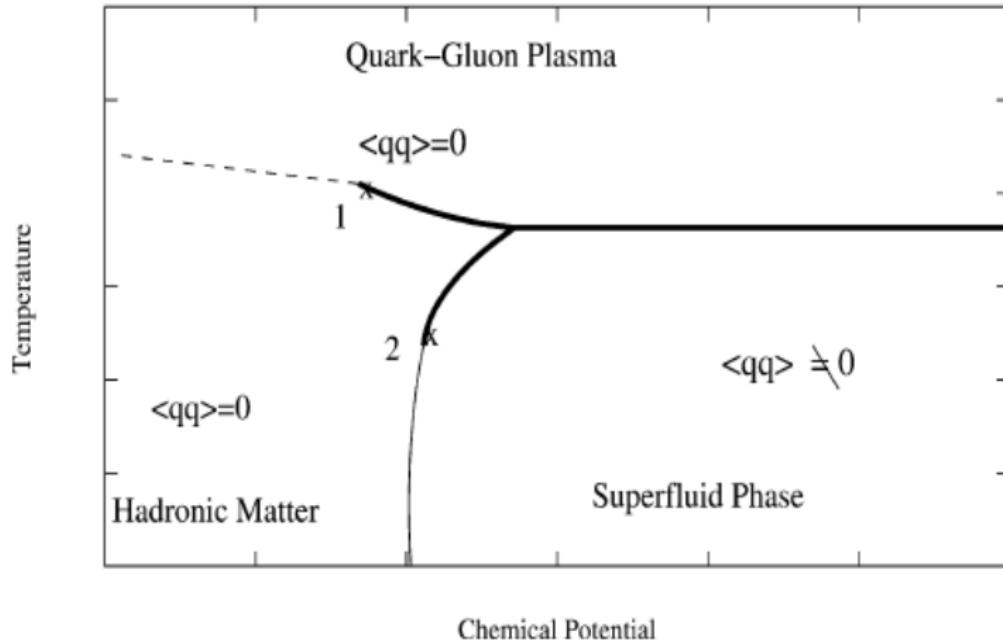
- ▶ Dense SU(2) QCD can shed light on dense SU(3) QCD
  - ▶ Calculation of different observables
  - ▶ Study of different physical phenomena
- ▶ Lattice study of SU(2) QCD contains full dynamics of real system (contrary to phenomenological models)

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The aim: numerical study of dense SU(2) QCD within lattice simulation

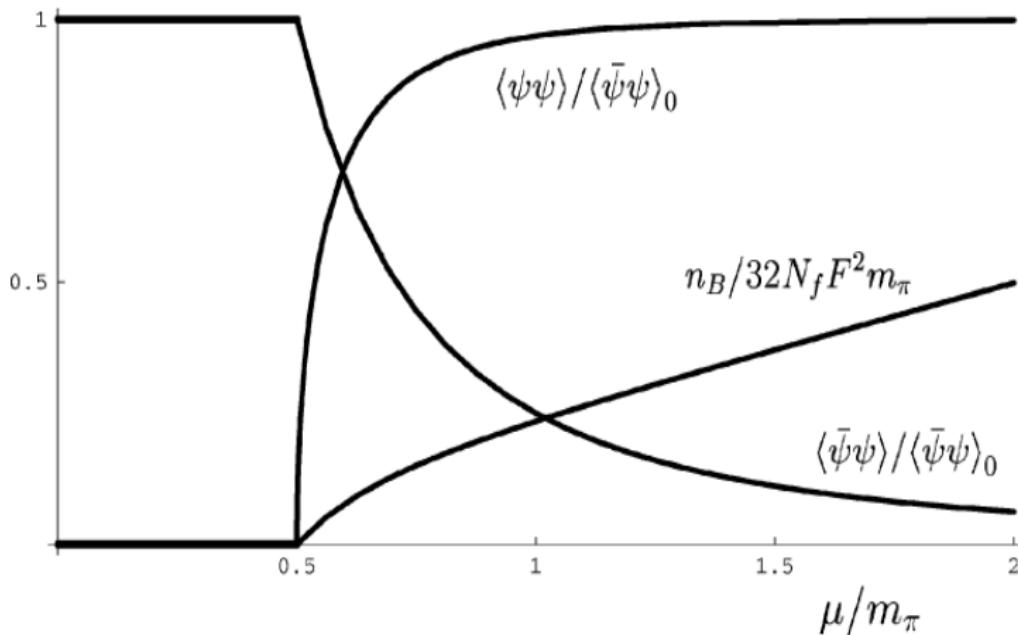
## Staggered fermions $N_f = 4$



J.B. Kogut, D. Toublan, D.K. Sinclair, Nucl. Phys. B 642 (2002) 181–209

One can build ChPT for SU(2) QCD

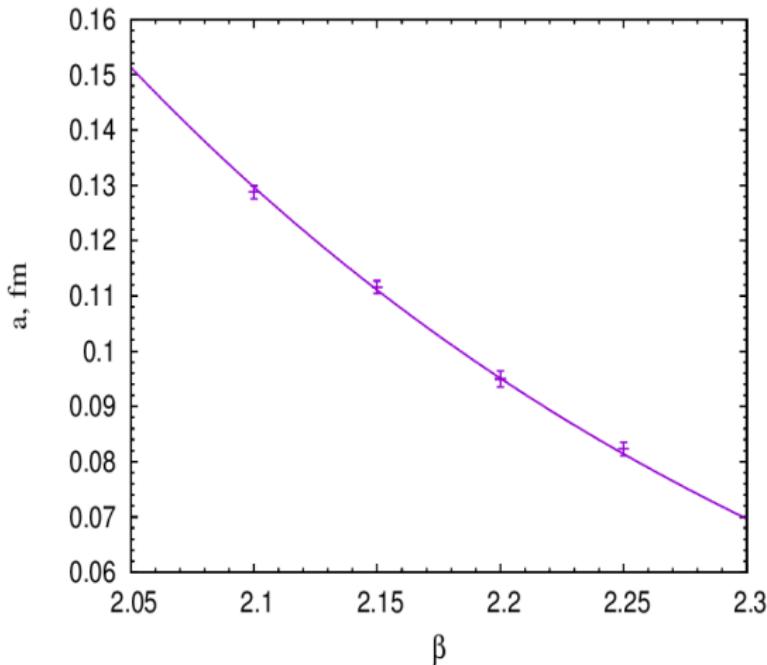
### Predictions of ChPT



## Details of the simulation:

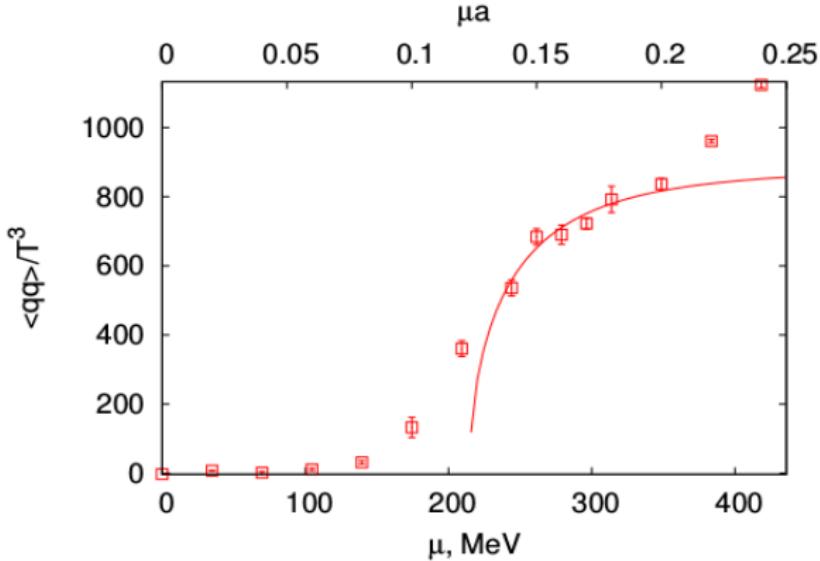
- ▶ Staggered fermions with rooting:  $N_f = 2$
- ▶ Lattice  $16^3 \times 32$ ,  $a = 0.11$  fm,  $m_\pi = 362(4)$  MeV,
- ▶ Diquark source in the action  $\delta S \sim \lambda \psi^T (C\gamma_5) \times \sigma_2 \times \tau_2 \psi$
- ▶ The symmetry breaking is different
  - ▶ Continuum:  $SU(2N_f) \rightarrow Sp(2N_f)$
  - ▶ Staggered fermions:  $SU(2N_f) \rightarrow O(2N_f)$
- ▶ Correct symmetry is restored in continuum limit
  - ▶ Naive limit  $a \rightarrow 0$ : two copies of  $N_f = 2$  fundamental fermions
  - ▶ Correct  $\beta$  function for  $a < 0.17$  fm
- ▶ See arXiv:1701.04664 by L. Holicki, J. Wilhelm, D. Smith, B. Welleghausen, L. von Smekal for detailed discussion

Beta function ( $\beta = \frac{4}{g^2}$ )



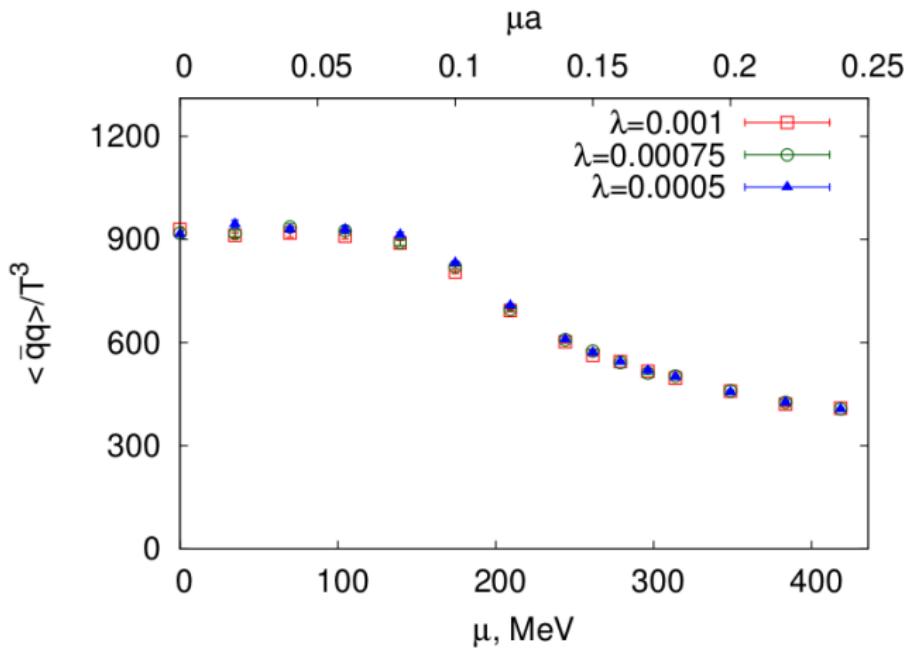
**Small chemical potential**  
 $\mu < 350 \text{ MeV}$

## Diquark condensate



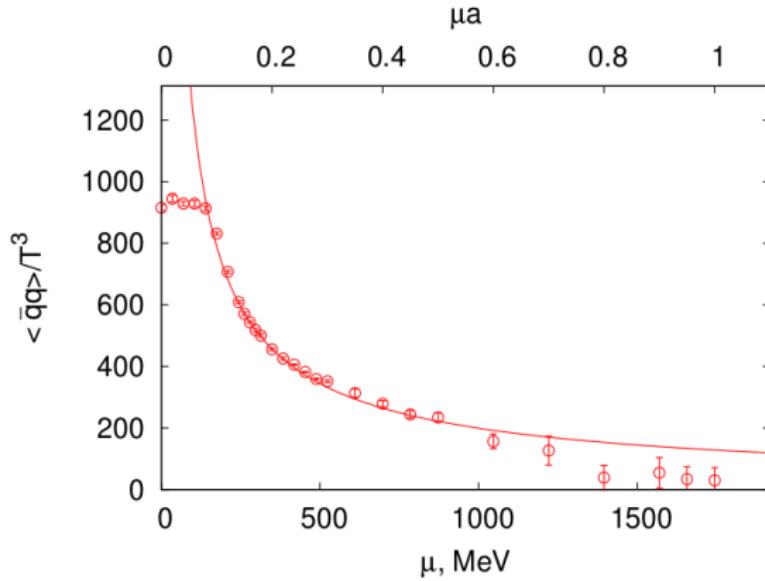
- ▶ Good agreement with ChPT  $\langle \psi\psi \rangle / \langle \bar{\psi}\psi \rangle_0 = \sqrt{1 - \frac{\mu_c^4}{\mu^4}}$
- ▶ Phase transition at  $\mu_c \sim m_\pi/2$
- ▶ Bose Einstein condensate (BEC) phase  $\mu \in (200, 350)$  MeV

## Chiral condensate



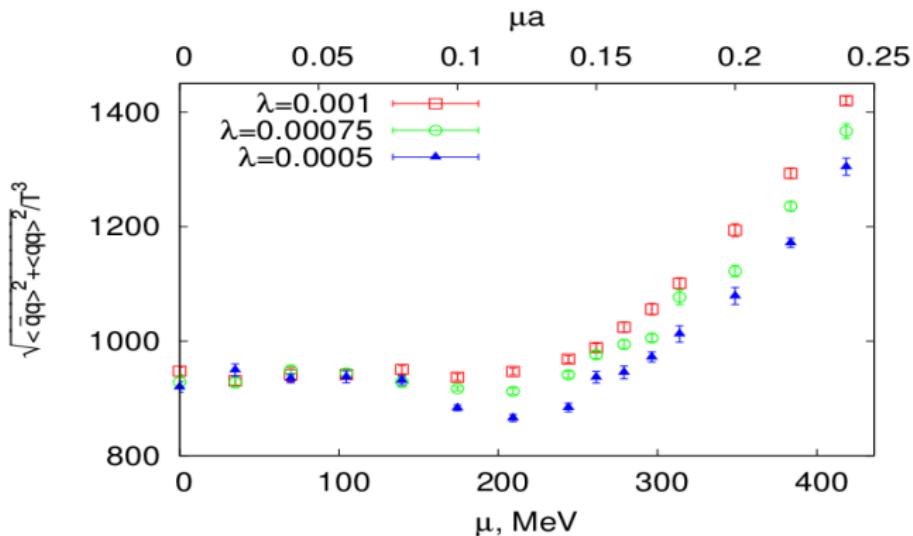
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## Chiral condensate



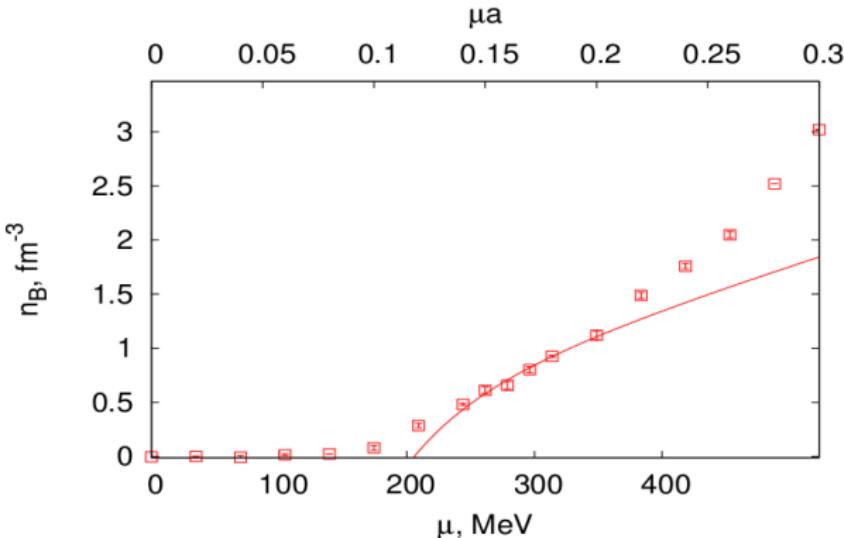
- ▶ ChPT prediction  $\langle \bar{\psi}\psi \rangle \sim \frac{m_\pi^2}{\mu^2}$
- ▶ We observe  $\langle \bar{\psi}\psi \rangle \sim \frac{1}{\mu^\alpha}, \alpha \sim 0.6 - 1.0$

## Circle relation



Circle relation:  $\langle \bar{\psi}\psi \rangle^2 + \langle \psi\psi \rangle^2 = const$

## Baryon density



- ▶ Good agreement with ChPT  $n \sim \mu - \frac{\mu_c^4}{\mu^3}$
- ▶ Phase transition at  $\mu_c \sim m_\pi/2$
- ▶ Deviations from ChPT prediction start from  $n \sim 1 \text{ fm}^{-3}$

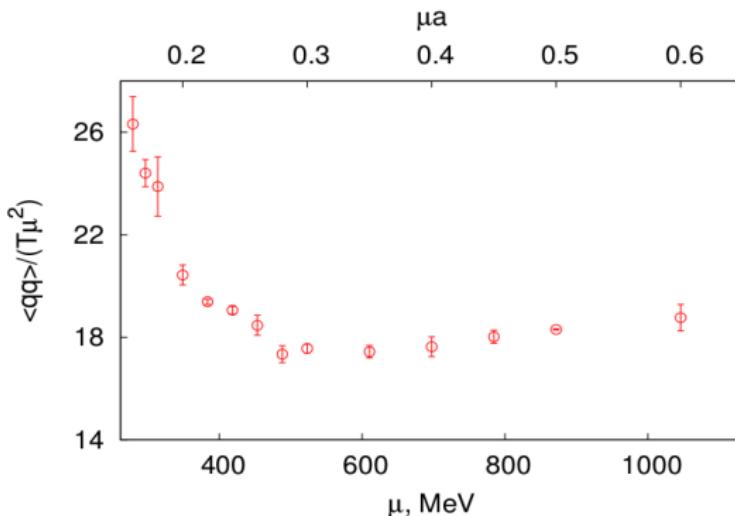
Large chemical potential  
 $\mu > 350$  MeV

## Phase diagram for $N_c \rightarrow \infty$

(L. McLerran, R.D. Pisarski, Nucl.Phys. A796 (2007) 83-100)

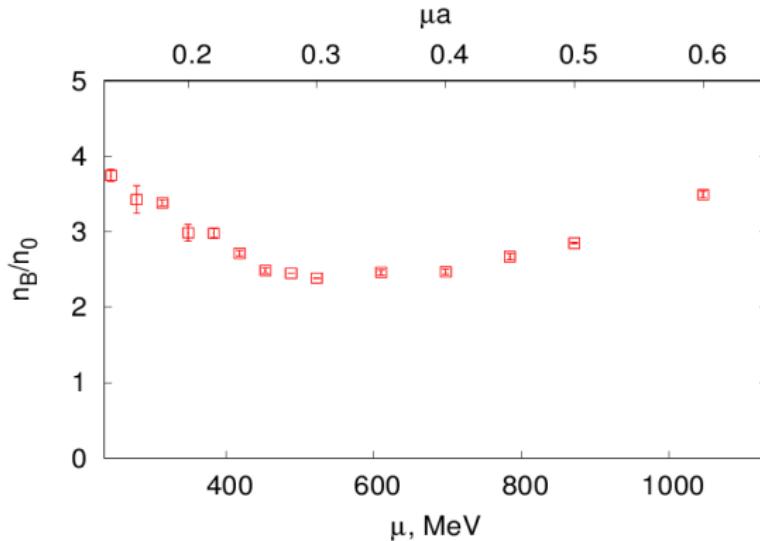
- ▶ Hadron phase  $\mu < M_N/N_c$  ( $p \sim O(1)$ )
- ▶ Dilute baryon gas  $\mu > M_N/N_c$  (width  $\delta\mu \sim \frac{\Lambda_{QCD}}{N_c^2}$ )
- ▶ Quarkyonic phase  $\mu > \Lambda_{QCD}$  ( $p \sim N_c$ )
  - ▶ Degrees of freedom:
    - ▶ Baryons (on the surface)
    - ▶ Quarks (inside the Fermi sphere  $|p| < \mu$ )
  - ▶ No chiral symmetry breaking
  - ▶ The system is in confinement phase
- ▶ Deconfinement ( $p \sim N_c^2$ )

## Diquark condensate



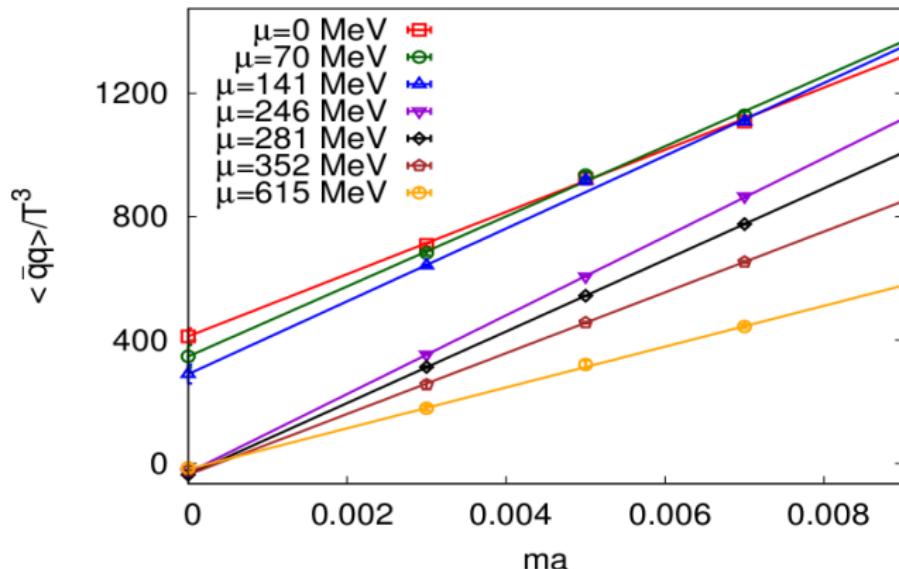
- ▶ Bardeen–Cooper–Schrieffer (BCS) phase  $\mu > 500$  MeV,  
 $\langle \psi\bar{\psi} \rangle \sim \mu^2$
- ▶ Baryons (on the surface)

## Baryon density



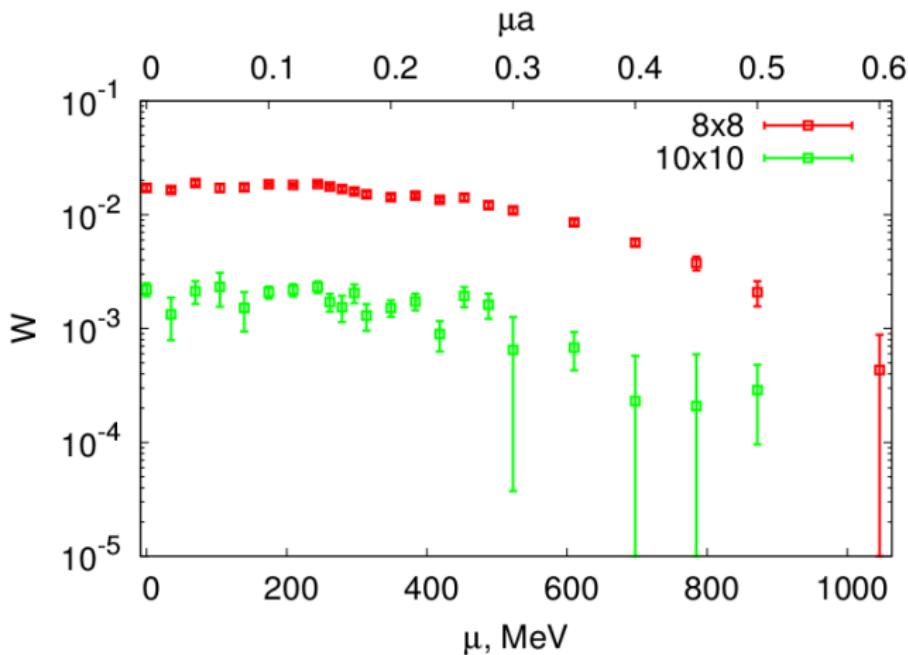
- ▶ Free quarks  $n_0 = N_f \times N_c \times (2s + 1) \times \int \frac{d^3 p}{(2\pi)^3} \theta(|p| - \mu) = \frac{4}{3\pi^2} \mu^3$
- ▶ **Quarks inside Fermi sphere**
- ▶ Quarks inside Fermi sphere dominate over the surface:  
 $\frac{4}{3}\pi\mu^3 > 4\pi\mu^2\Lambda_{QCD} \Rightarrow \mu > 3\Lambda_{QCD}$  ( $n \sim (5 - 10) \times \text{nuclear density}$ )

## Chiral condensate (chiral limit $m \rightarrow 0$ )



Chiral symmetry is restored

## Wilson loop



Polyakov loop is zero. The system seems to be in the confinement phase

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We observe quarkyonic phase

## Conclusion:

- ▶ We observe  $\mu < m_\pi/2$  hadronic phase
- ▶ Transition to superfluid phase  $\mu \simeq m_\pi/2$  (BEC)
- ▶  $\mu > m_\pi/2, \mu < m_\pi/2 + 150$  MeV dilute baryon gas
- ▶ Hadronic phase and BEC phase are well described by ChPT
- ▶ Deviation from ChPT from  $\mu > 350$  MeV (dense matter)
- ▶ BCS phase  $\mu \sim 500$  MeV, transition BEC→BCS is smooth
- ▶ BCS phase is similar to quarkyonic phase

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## Predictions for SU(3) (estimation!):

- ▶ Quarkyonic phase starts from  $n \sim (5 - 10) \times$  nuclear density
- ▶ Restoration of chiral symmetry  $((5 - 10) \times$  nuclear density)  $\Rightarrow$  can be seen in experiment

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Outlook: simulations with improved action at larger  $\mu$  in search of deconfinement